
Grade 3

Mathematics Formative Assessment Lesson

Designed by Kentucky Department of Education Mathematics Specialists to be Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues, please contact the KDE team at: kdemath@education.ky.gov

Created for the sole purpose of assisting teachers as they develop student understanding of Kentucky’s Core Academic Standard through the use of highly effective teaching and learning.

Not intended for sale.
Attributes of Shapes

Mathematical goals
This lesson unit is intended to help you assess how well students are able to:
• Identify defining attributes of shapes
• Think abstractly and concretely
• Defend mathematical decisions

Common Core State Standards
This lesson involves a range of mathematical practices from the standards, with emphasis on:
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
6. Attend to precision
7. Look for and make use of structure

This lesson asks students to apply mathematical content from:
• 3.G – Reason with shapes and their attributes

Introduction
In this unit, students use the attributes of shapes to view shapes as members of both specific sub-categories and larger, more general, categories.

• Before the lesson, students attempt the task Attributes of Shapes Pre-Lesson Assessment individually. You then review their work, and create questions for students to answer in order to improve their solutions.
  o Questions created may be in response to individual student work, or collective need.
• At the start of the lesson portion, share the questions you created in order to frame the cooperative learning time.
• Next, students work collaboratively in small groups. Their task is to sort shapes into categories with common attributes, and identify the larger categories from which the subcategories “came from”.
• In a whole-class discussion, students compare and evaluate the mathematical arguments they have seen and used.

Materials required
• The Attributes of Shapes Pre-Lesson Assessment for each student.
• Each individual student will need a copy of the task sheet Shapes which should already be cut up and neatly, packaged for students.
• One legal size paper for each pair of students.
• *Extension Cards* (not required, but should be available to differentiate for students who find the challenge level of the *Shapes* task too easy)

**Time needed**
Approximately fifteen minutes before the lesson and a one-hour lesson. Timings given are only approximate. Exact timings will depend on the needs of your class.

**Before the lesson**

**Assessment task: Attributes of Shapes Pre-Lesson Assessment**
Have the students do this task in class a day or more before the lesson. This will give you an opportunity to assess their work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

It is important that students answer the questions without assistance, as far as possible. If students are struggling to get started, ask questions that help them understand what they are being asked to do, but do not do the problem for them. See the *Common Issues* table on the following page.

Students should not worry too much if they cannot understand or do everything, because soon there will be a lesson using the same task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

**Assessing students’ responses**
Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their problem solving strategies.

We suggest that you do not score students’ work. The research shows that this will be counterproductive, as it will encourage students to compare scores, and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given the *Common Issues* table on the next page. We suggest that you make a list of your own questions, based on your students’ work, using the ideas on the following page. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.
## Suggested questions and prompts:

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<th>Common issues:</th>
<th>Suggested questions and prompts:</th>
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| **Students have a difficult time getting started.**                          | • What are the directions? Read them again please.  
• Choose four or five shapes. Within this smaller set, do these shapes share any attributes? |
| **Students don’t seem to be aware of the attributes of shapes they might consider when sorting.** | • What kind of shape is this? Draw another (identified shape). What qualities or attributes make this shape a (the identified shape)?  
• What kinds of attributes define shapes? For example, in what ways is a quadrilateral different from a triangle? A rhombus from a square?  
• Does the fact that this (point to a smaller triangle) is smaller than this (point to a larger triangle), mean that these two shapes belong to different categories? What is the same, and what is different about these two shapes? |
| **Students aren’t applying their knowledge of defining attributes to sort shapes in ways that are logical.** | • What is an attribute that (identify a shape) this shape has? Write that down on a scrap piece of paper and use that as a heading to help group other shapes. |
| **Students have difficulty identifying the common attributes of shapes.**     | • What are the defining attributes of shapes? Let’s write them down. Do any of these shapes share these attributes? |
| **Students have difficulty finding attributes that would allow a class to be divided into subclasses.** | • What attribute is it that all of these shapes have in common? Are there any defining attributes that are different among the shapes you have identified?  
• Have you tried making a list of the attributes you’ve already investigated? May I see it?  
• What is the same, and what is different about these two shapes? |
| **Students find the initial task/lesson task too easy.**                      | • Distribute the set of Extension Cards. |

### Suggested lesson outline

**Introduction and individual work (10 minutes)**
Display or distribute the list of questions you created in response to student work on the *Attributes of Shapes Pre-Lesson Assessment*. 
[Last lesson] you worked on … Do you recall what the task was about? I have read your solutions, and I have some questions I’d like you to think about during today’s cooperative learning time. These questions will frame today’s lesson and learning.

**Collaborative small-group work (20 minutes)**
Organize students into homogeneous pairs based on common misconceptions from the *Attributes of Shapes Pre-Lesson Assessment*.

*Today, you and your partner are going to sort shapes according to defining attributes. You are also going to discuss some of the attributes shared by various shapes.*

*Things other than shapes have attributes, such as people, students, and animals. For example, all of you share an attribute because you are all students in this class. How might we use some of the different attributes you have to organize you into smaller groups?*

Record students’ ideas in order to create a piece of work that the class can model their own work from that resembles the organization below:

*Students, you and your partner should organize your work similar to the organizer I made for our class, using labels and branches. You may sort your shapes into any categories you like, as long as all the shapes in any category share the defining attribute of that category.*
Give each pair a set of *Shapes* and 1 piece of legal-sized paper.

You have two roles while students are working, to monitor student methods, deciding which to select and use during the whole class discussion, and to support student problem solving.

**Find out about student methods**
As students continue to sort shapes into categories and sub-categories, circulate about the room; the purpose of circulating is to listen and monitor student methods/arguments. Use the information you gleaned from the initial, individual task to decide where to focus your attention.

Research indicates that teachers who have thoughtfully considered anticipated student responses are better able to address those misconceptions if they occur; moreover, these teachers are better able to address student misconceptions when/if they arise.

- Try to determine if students are making progress beyond their initial work, and if they aren’t remind them of the feedback you have provided to them, either via whole class feedback or individual feedback.
- Are students able to reconcile the approach(es) of their partners to their own work, if the partner’s work is different?
- Are students organizing their work in a systematic way, keeping track of insights as they arise?
- Have students tried eliminating possibilities from categories?
- Once students arrive at a comfortable solution, do they reflect on their approach and communicate the approach, conclusion, and reasoning in a mathematically precise way?

**Support student problem solving**
Try not to prompt students into using a particular sorting scheme, and try not to point out the difficulties with their chosen methods to them. Instead, ask questions to prompt students to justify, and evaluate their own sorting.

The questions in the *Common Issues* table might guide your questioning, as might the questions in the preceding section.

If any group finishes their solution early, or if a group finds the initial task too easy, provide them with the *Extension Cards* - task them with determining where the extension cards would fit into their organizer and identify any congruent or similar to the any of the first set of shapes.

Once most groups have finished with the initial card sort, structure a pair-square, in which the original pair of students pairs up with another pair, making a “square” of
four students. Invite the two pairs to share the ways the sorted the shapes with one another.

**Plenary whole-class discussion: comparing different approaches (10 minutes)**

Organize a whole-class discussion of the *Shapes* task. Focus the discussion on the methods students have seen and used during the lesson, rather than discussing who has the “best” argument or a “correct” sorting.

Some of the following questions may help you organize the whole-class discussion. These questions are to be investigated by you prior to the discussion:

- Did some pairs “skip” categories, or go directly to the specifics?
- At any branch point, do any of the stemming categories fit into any of the other stemming categories? For example, if a student went from QUADRILATERALS to SQUARES, RECTANGLES, and RHOMBI then attention needs to be given to the attributes these three stemming share and do not share.
- What was the smallest group to which a shape belonged?

Try to avoid making evaluative comments yourself. Instead, encourage students to respond to other students’ explanations.

If some of your students were given the *Extension Cards* save their sharing for last.

**Review individual solutions to the assessment task (10 minutes)**

Ask students to reread their original solutions, and write about what they have learned during the lesson.
1. Some of these shapes have attributes in common that we have learned about in the past. Identify one attribute that at least three of the shapes share. Write down the attribute and the shapes that have this property.

2. Do the same thing again, but this time, choose a different attribute. It’s ok if you want to reuse any of the shapes from the first problem.
3. Find and identify at least two other shapes in the original assortment that share at least one attribute with this shape. Which attribute do they share?
Attributes of Shapes – *Shapes Cards*
Attributes of Shapes – *Extension Cards*

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