Mathematical Goals

This lesson is intended to evaluate the understanding of your students in multiplication and how well students are able to interpret various representations of multiplication facts. It will help you to identify students who have the following difficulties:

- lack of conceptual understanding of the properties of numbers
- do not see connections from addition to multiplication
- have multiplication facts memorized, but lack conceptual understanding

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

**Operations and Algebraic Thinking** 3.OA
- Represent and solve problems involving multiplication and division.
- Multiply and divide within 100.

This lesson involves a range of *Standards for Mathematical Practices*, with emphasis on:

1. Make sense of problems and persevere in solving them.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. The teacher then reviews their work and creates questions for students to answer in order to improve their solutions.
- Students work in small groups on a collaborative discussion task matching multiple representations of multiplication problems. Throughout their work, students justify and explain their decisions to their peers.
- Teacher leads a whole class discussion around the findings and responses of the groups.
- Students return to their original assessment tasks, and try to improve their own responses.
Materials required

Each individual student will need:
- Two copies of the assessment task *Multiplication*. (One for the initial task and one for the revision.)

Each small group of students will need the following resources:
- Card Set A
- Have Card Set B ready, but do not pass out.
- Have Card Set C ready, but do not pass out.
- Have Card Set D ready, but do not pass out.
- Have Card Set E ready, but do not pass out.

Time needed

Approximately 15 minutes before the lesson (for the individual assessment task), one 40 minute lesson (30 minutes for group task and 10 minutes for whole class discussion), and 15 minutes for a follow-up lesson (for students to revisit individual assessment task). Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the Lesson

**Assessment task: Multiplication (15 minutes)**

Students should have already been exposed to lessons that required them to build, draw, and write story problems for multiplication. Work should be focused around repeated addition, area models, use of arrays, equal groups, and story problems.

Have students do this task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with conceptual understanding of multiplication. You will be able to target your help more effectively in the follow-up lesson. Depending on your class you can have them do it all at once or in small groups (they should still work individually).

Give each student a copy of the assessment task *Multiplication*.

*Using the multiplication problem at the top of each chart, create a model/representation of the problem that matches the labels in each of the four boxes.*

It is important that the students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.
**Assessing students’ responses**
Collect students’ responses to the task. Make some notes about what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score student’s work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions on the following page may serve as examples. These questions have been drawn from commonly identified student misconceptions.

We suggest that you write a list of your own questions, based on your students’ work, using the ideas that follow. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson before students revisit initial task.
Below is a list of common issues and questions/prompts that may be written on individual initial tasks or during the collaborative activity to help students clarify and extend their thinking.

<table>
<thead>
<tr>
<th>Common Issues:</th>
<th>Suggested questions and prompts:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A group has trouble getting started.</td>
<td>• What information do you already know?</td>
</tr>
<tr>
<td></td>
<td>• What do you need to find out?</td>
</tr>
<tr>
<td>Students draw different representations for the</td>
<td>• You may choose to compare these two cards in whole group discussion.</td>
</tr>
<tr>
<td>missing cards. For example one child may draw 3</td>
<td>• Which card is correct?</td>
</tr>
<tr>
<td>groups of six while other students may draw 6 groups</td>
<td>• What property of multiplication is this representative of?</td>
</tr>
<tr>
<td>of 3.</td>
<td></td>
</tr>
<tr>
<td>Students change orientation of the array model.</td>
<td>• How is this model different if we turn it on its side?</td>
</tr>
<tr>
<td></td>
<td>• Is the orientation of the model important?</td>
</tr>
<tr>
<td></td>
<td>• How does this link to the property of commutative property of multiplication?</td>
</tr>
<tr>
<td>Students are not attentive to details and structure</td>
<td>• Provide manipulatives for students.</td>
</tr>
<tr>
<td>of the word problems.</td>
<td>• Can you draw a model?</td>
</tr>
<tr>
<td></td>
<td>• Can you act out what the problem is telling us?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Suggested lesson outline

Collaborative Activity: Multiplication Card Sort
Strategically organize the class into groups of two or three students based on the data collected from the initial task results. With larger groups, some students may not fully engage in the task.

Introduce the lesson carefully:

*I want you to work together with your partner/group. You are going to work on matching cards that represent the same math idea.*

Have cards ready to show the students a model of two cards that would match and two cards that would not match. Have a student tell why the cards do/do not match.

Every group of students may not work through all card sets. Groups should work at their own pace.

During the small group work, the teachers’ tasks are:

- to question but not interfere with student work.
- to make notes of student approaches to the task.
- to support student problem solving through guided questions.

Make notes of student approaches to the task
You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may not consider or understand the concept of multiplication. Students may be able to recall the facts, but have difficulty attaching the meaning to a model. This lack of understanding prevents students from applying multiplication in real life situations.

Support student problem solving
Try not to make suggestions that move students toward a particular approach to the task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

When a student creates a match, challenge their partner to provide an explanation.

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a whole class discussion.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations.

Card Set A
*Note: There will be one expression that does not have a match. However, there is a blank card provided for the students to draw their own representation to complete the set.*
Card Set B
As students finish with Card Set A and are able to explain their reasoning give them Card Set B. Do not take up the previous sets of cards. Students may use these for guidance in making further decisions. Card Set B will consist of area model representations.

Card Set C
As students finish with Activity B and are able to explain their reasoning give them Card Set C. Card Set C will consist of repeated addition representations. *Note: There will be one blank card for students to fill in to complete this level. Do not tell students; allow them to develop this idea on their own.

Card Set D
As students finish with Activity C and are able to explain their reasoning give them Card Set D. Card Set D is the “sets of” cards. *Note: There will be one blank card in this set in which the students will need to fill in to complete the set. Do not tell students; allow them to develop this idea on their own.

Card Set E
As students finish with Card Set D and are able to explain their reasoning give them Card Set E. Card Set E will consist of interpreting word problems. There are distractor cards in this set that do not have a match. These are to promote interpretation of the word problems in context.

Sharing Work (10 minutes)
When students get as far as they can with the task, ask one student from each group to visit another group’s work. Students remaining at their desk should explain their reasoning for the way they worked the problem at their own desk.

*If you are staying at your desk, be ready to explain the reasons for your group’s work.
If you are visiting another group, check to see which answers or explanations are different from your own.

If their matches are different than your groups, ask for an explanation. If you still don’t agree, explain your own thinking. When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.

Provide time for groups to discuss and make changes to their original work.

Extension activities
Students may create their own set of cards with different multiplication facts. If time allows, they may challenge each other to sort their sets.

Plenary whole-class discussion (10 minutes)
Choose students to share how they figured out which model matched the expressions. Discuss which model they liked best and why.
Conclude the lesson by discussing and generalizing what has been learned. The generalization involves extending what has been learned to new examples.

**Improving individual solutions to the assessment task (10 minutes)**

Return to the students their original assessment, *Multiplication*, as well as a second blank copy of the task.

> Look at your original responses and think about what you have learned during this lesson. Using what you have learned, try to improve your work.

If you (teacher) have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

*Card sorting tasks adapted from [www.makingmathmagic.com](http://www.makingmathmagic.com)*
Using the multiplication problem at the top of each chart, create a representation of the problem that matches the labels in each of the four boxes.

<table>
<thead>
<tr>
<th>4 x 7 = 28</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Area Model</td>
<td>Equal Groups</td>
</tr>
<tr>
<td>Repeated Addition</td>
<td>Word Problem</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7 x 8 = 56</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Area Model</td>
<td>Equal Groups</td>
</tr>
<tr>
<td>Repeated Addition</td>
<td>Word Problem</td>
</tr>
</tbody>
</table>
Card Set A

3 x 4

4 x 8

6 x 3

4 x 9

7 x 8

5 x 3

Card sets adapted from Making Math Magic  http://makingmathmagic.com/
Card Set B

Card sets adapted from Making Math Magic  http://makingmathmagic.com/
### Card Set C

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4 + 4 + 4</td>
<td>8 + 8 + 8 + 8</td>
</tr>
<tr>
<td>3 + 3 + 3 + 3 + 3 + 3</td>
<td></td>
</tr>
<tr>
<td>8 + 8 + 8 + 8 + 8 + 8 + 8</td>
<td>3 + 3 + 3 + 3 + 3</td>
</tr>
</tbody>
</table>

Card Set D

3 sets of 4

6 sets of 3

4 sets of 9

7 sets of 8

5 sets of 3

Card sets adapted from Making Math Magic  http://makingmathmagic.com/
## Card Set E

<table>
<thead>
<tr>
<th>Connlee walked for 3 hours. He walked at the rate of 4 miles per hour. How many miles did Connlee walk?</th>
<th>Sarah has 8 bracelets. Madison has 4 times as many bracelets as Sarah. How many bracelets does Madison have?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noah made 6 shots in his basketball game. Each shot was worth 3 points. How many points did Noah score?</td>
<td>Noah bought 4 packs of juice. Each pack had nine bottles of juice in it. How many bottles of juice did Noah buy?</td>
</tr>
<tr>
<td>Joanna planted 7 rows of trees. She planted 8 trees in each row. How many trees did Joanna plant?</td>
<td>Katrina has 3 oranges. Renee has five times as many oranges as Katrina. How many oranges does Renee have?</td>
</tr>
</tbody>
</table>
### CARD SET E continued

<table>
<thead>
<tr>
<th>Question 1</th>
<th>Question 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connlee walked for 3 hours on Monday. He walked for 4 more hours on Tuesday. How many total miles did Conlee walk?</td>
<td>Sarah has 8 bracelets. She gave four of her bracelets to Madison. How many bracelets does Sarah have left?</td>
</tr>
<tr>
<td>Noah made 6 shots in his first basketball game and three shots in his next basketball game. How many shots did Noah make in both games together?</td>
<td>Noah bought 9 bottles of juice and drank 4 of them this week during lunch. How many bottles of juice does Noah have left?</td>
</tr>
<tr>
<td>Joanna planted 7 rows of peach trees and 8 rows of apple trees. How many rows of trees did Joanna plant?</td>
<td>Katrina has 3 oranges. She went to the store and bought five oranges. How many oranges does Katrina have altogether?</td>
</tr>
</tbody>
</table>