



# CCGPS Frameworks Student Edition

## Mathematics

7<sup>th</sup> Grade

Unit 1: Operations with Rational Numbers



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*"Making Education Work for All Georgians"*

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**Unit 1**  
**Operations with Rational Numbers**

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## OVERVIEW

In this unit students will:

- apply and extend previous understandings of addition and subtraction to add and subtract rational numbers.
- represent addition and subtraction on a horizontal or vertical number line diagram.
- describe situations in which opposite quantities combine to make 0.
- understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative.
- show that a number and its opposite have a sum of 0 (are additive inverses).
- interpret sums of rational numbers by describing real-world contexts.
- understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ .
- show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- apply properties of operations as strategies to add and subtract rational numbers.
- apply and extend previous understandings of multiplication and division to multiply and divide rational numbers.
- understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers.
- interpret products of rational numbers by describing real-world contexts.
- understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.
- understand if  $p$  and  $q$  are integers then  $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$ .
- interpret quotients of rational numbers within real-world contexts.
- apply properties of operations as strategies to multiply and divide rational numbers.
- convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.
- solve real-world and mathematical problems involving the four operations with rational numbers.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the

appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

## STANDARDS ADDRESSED IN THIS UNIT

### KEY STANDARDS

**Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.**

**MCC7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

**MCC7.NS.1a** Describe situations in which opposite quantities combine to make 0.

**MCC7.NS.1b** Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

**MCC7.NS.1c** Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

**MCC7.NS.1d** Apply properties of operations as strategies to add and subtract rational numbers.

**MCC7.NS.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.

**MCC7.NS.2a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

**MCC7.NS.2b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers then  $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$ . Interpret quotients of rational numbers by describing real-world contexts.

**MCC7.NS.2c** Apply properties of operations as strategies to multiply and divide rational numbers

**MCC7.NS.2d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.

**MCC7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

## **STANDARDS FOR MATHEMATICAL PRACTICE**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### **1. Make sense of problems and persevere in solving them.**

In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”

### **2. Reason abstractly and quantitatively.**

In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

### **3. Construct viable arguments and critique the reasoning of others.**

In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?”. They explain their thinking to others and respond to others’ thinking.

**4. Model with mathematics.**

In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. *Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.*

**5. Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data.

**6. Attend to precision.**

In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. *Students use appropriate terminology when referring to rates, ratios, components of expressions, equations or inequalities.*

**7. Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e.  $6 + 2x = 3(2 + x)$  by distributive property) and solve equations (i.e.  $2c + 3 = 15$ ,  $2c = 12$  by subtraction property of equality),  $c=6$  by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume.

**8. Look for and express regularity in repeated reasoning.**

In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that  $a/b \div c/d = ad/bc$  and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

## **ENDURING UNDERSTANDINGS**

- Computation with positive and negative numbers is often necessary to determine relationships between quantities.

- Models, diagrams, manipulatives and patterns are useful in developing and remembering algorithms for computing with positive and negative numbers.
- Properties of real numbers hold for all rational numbers.
- Positive and negative numbers are often used to solve problems in everyday life.

## CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained:

- positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- rational numbers are points on the number line.
- numbers with opposite signs indicate locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite
- absolute value of a rational number is its distance from 0 on the number line
- interpret absolute value as magnitude for a positive or negative quantity in a real-world situation

## SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school children. **Note – At the middle school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary

<http://intermath.coe.uga.edu/dictionary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Additive Inverse:** The sum of a number and its additive inverse is zero.
- **Multiplicative Inverse:** Numbers are multiplicative inverses of each other if they multiply to equal the identity, 1.
- **Absolute Value:** The distance between a number and zero on the number line. The symbol for absolute value is shown in this equation:  $|-8| = 8$
- **Integers:** The set of whole numbers and their opposites  $\{\dots-3, -2, -1, 0, 1, 2, 3\dots\}$
- **Long Division:** Standard procedure suitable for dividing simple or complex multi-digit numbers. It breaks down a division problem into a series of easier steps.
- **Natural Numbers:** The set of numbers  $\{1, 2, 3, 4, \dots\}$ . Natural numbers can also be called counting numbers.
- **Negative Numbers:** The set of numbers less than zero.
- **Opposite Numbers:** Two different numbers that have the same absolute value. Example: 4 and -4 are opposite numbers because both have an absolute value of 4.
- **Positive Numbers:** The set of numbers greater than zero.
- **Rational Numbers:** The set of numbers that can be written in the form  $a/b$  where  $a$  and  $b$  are integers and  $b \neq 0$ .
- **Repeating Decimal:** A decimal number in which a digit or group of digits repeats without end.
- **Terminating Decimal:** A decimal that contains a finite number of digits.
- **Zero Pair:** Pair of numbers whose sum is zero.



## Models for Teaching Operations of Integers


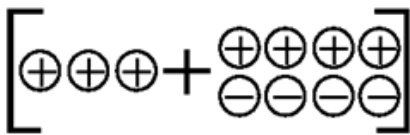
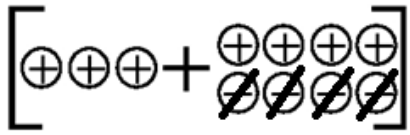

These models have been adapted from <http://teachers.henrico.k12.va.us/math/hcpsalgebra1/>.

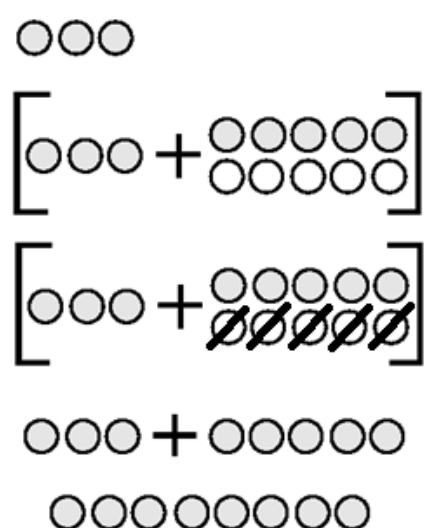
The following are some everyday events that can be used to help students develop a conceptual understanding of addition and subtraction of integers.

- Getting rid of a negative is a positive. For example: Johnny used to cheat, fight and swear. Then he stopped cheating and fighting. Now he only has 1 negative trait so...  
 $(3 \text{ negative traits}) - (2 \text{ negative traits}) = (1 \text{ negative trait})$  or  $(-3) - (-2) = (-1)$
- Using a credit card example can make this subtraction concept clearer. If you have spent money you don't have  $(-5)$  and paid off part of it  $(+3)$ , you still have a negative balance  $(-2)$  as a debt, or  $(-5) + 3 = (-2)$ .
- Draw a picture of a mountain, the shore (sea level) and the bottom of the ocean. Label sea level as 0.

Any of the following models can be used to help students understand the process of adding or subtracting integers. If students have trouble understanding and using one model you can show students how to use another model.

### 1. The Charged Particles Model (same as using two-color counters)

When using charged particles to subtract, $3 - (-4)$ for example, you begin with a picture of 3 positive particles.	
Since there are no negative values to "take away", you must use the Identity Property of Addition to rename positive 3 as $3 + 0$ . This is represented by 4 pairs of positive and negative particles that are equivalent to 4 zeros.	
Now that there are negative particles, you can "take away" 4 negative particles.	
The modeled problem shows that the result of subtracting 4 negative particles is actually like adding 4 positive particles. The result is 7 positive particles. This is a great way to show why $3 - (-4) = 3 + 4 = 7$	

<b>Two-Color Counters Method</b>	
<p>When using two-colored counters you would use the yellow side to represent positive integers and the red side to represent negative numbers.</p> <p>The problem represented is <math>-3 - 5</math>.</p>	 <p>The diagram illustrates the two-color counter method for the problem <math>-3 - 5</math>. It consists of four rows of counters:</p> <ul style="list-style-type: none"> <li>Row 1: Three positive counters (represented by circles).</li> <li>Row 2: A set of three positive counters and five negative counters (represented by circles with diagonal lines) enclosed in large square brackets, with a plus sign between them.</li> <li>Row 3: The same set of three positive counters and five negative counters in brackets, but the five negative counters are crossed out with diagonal lines.</li> <li>Row 4: Eight positive counters, representing the final result of the operation.</li> </ul>

## 2. The Stack or Row Model

<p>To model positive and negative integers, use colored linking cubes and graph paper. Graph paper and colored pencils will allow students to record problems and results. Students should also write the problems and answers numerically.</p>	
<p>Create stacks or rows of numbers with the colored linking cubes and combine/compare the cubes. If the numbers have the same sign, then the cubes will be the same color. Stress that adding is like <i>combining</i>, so make a stack or row to show this.</p>	<p style="text-align: center;"><math>(-3) + (-4) = (-7)</math></p>
<p>If the numbers are not the same sign (color), for example <math>-3 + 5</math>, you compare the stacks of different colors. Using the concept of zero pairs, the result is the difference between the stacks or the result is based on the higher stack. This is easy to see and understand.</p>	
<p>For subtraction you create zeros by pairing one of each color. Then add as many zeros to the first number as needed so that you can take away what the problem calls for. Now physically take away the indicated amount and see what is left. The example problem shown is <math>3 - (-4)</math>.</p>	

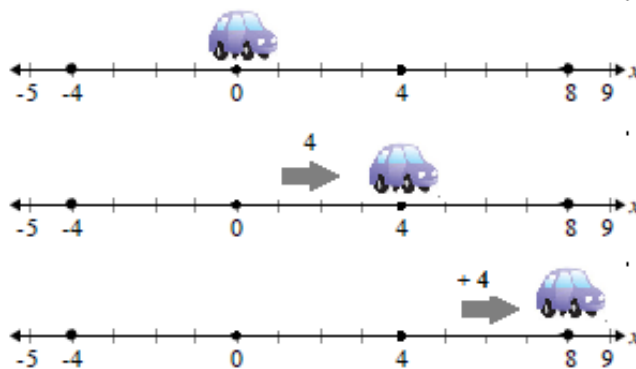
## 3. The Hot Air Balloon Model

<p>Sand bags (<i>negative integers</i>) and Hot Air bags (<i>positive integers</i>) can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted).</p>	
<p>Here is an example: <math>-3 - (-4) = ?</math></p> <ul style="list-style-type: none"> <li>• The balloon starts at <math>-3</math> (think of the balloon being 3 feet below sea level or 3 feet below the level of a canyon) and you take off 4 sand bags.</li> <li>• Now, think about what happens to a balloon if you remove sand bags, the balloon gets lighter. So, the balloon would go up 4 units.</li> </ul> <p>If you think in terms of a <i>vertical number line</i>, it would start at <math>-3</math> and end up at <math>1</math>, so <math>-3 - (-4) = 1</math>. To help students make the connection between <math>-3 - (-4)</math> and <math>-3 + (+4)</math>, present the addition and subtraction questions using the same numbers.</p>	
<p>Another example would include the first addition question as <math>9 + (-5)</math> and the first subtraction question would then be <math>9 - (+5)</math>. The students see that <i>putting on</i> 5 sand bags (negative) produces the same result as <i>taking off</i> 5 hot air bags (positive).</p>	

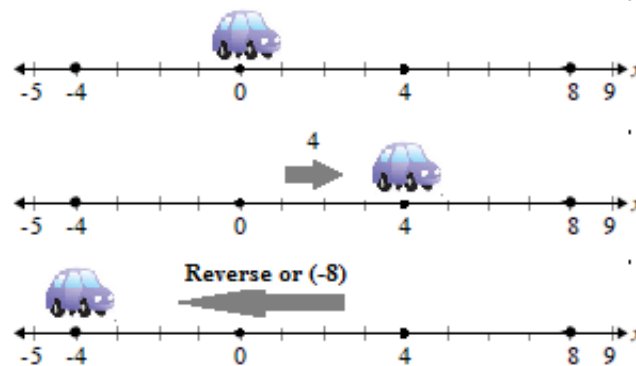
#### 4. The Number Line Model

You can describe addition and subtraction of integers with a number line and a toy car. The car faces forward (to the right) to represent a positive direction. The car is moved forward to represent a positive integer. The car flips around backward (facing left) to represent a negative direction or subtraction. The car is moved backward (reverse) to represent a negative integer.

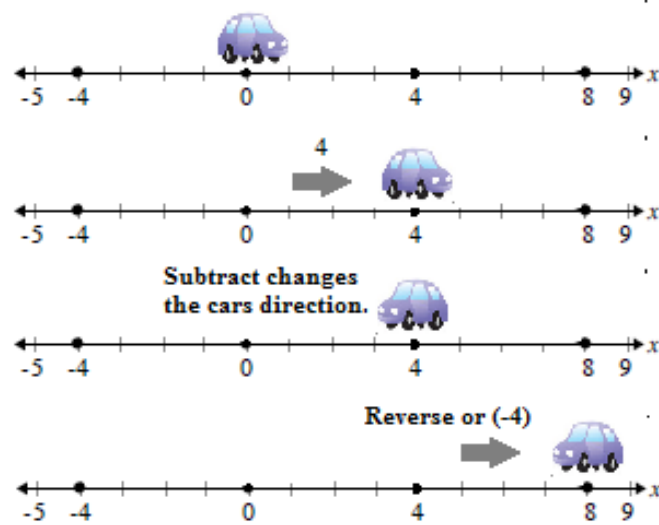
Example 1:  $4 + 4 = 8$



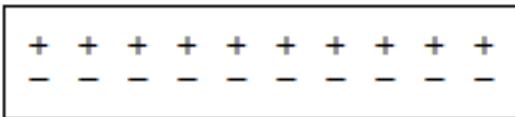
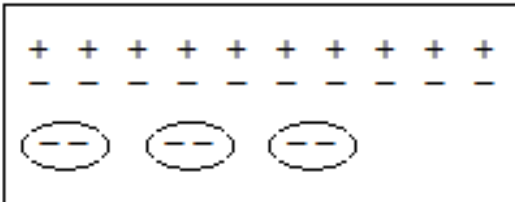
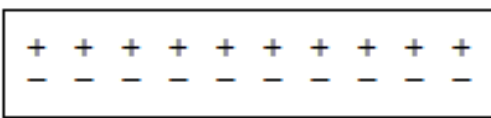

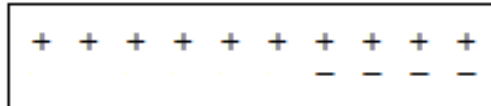
Example 2:  $4 + (-8) = -4$



Example 3:  $4 - (-4) = 8$



### 5. Charged Particle Model for Multiplication

The charged particle method can be used to illustrate multiplication of integers.	
<p>To begin, a model with a 0 charge is illustrated. The 0 charge model will allow us to work with positive and negative integers.</p> <p><u>Example 1:</u> In this problem, <math>3 \times (-2)</math>, three groups of two negative charges is added to the 0 charged field. The result is <math>(-6)</math>.</p>	<p><b>0 Charge</b></p>  <p><b><math>3 \times (-2)</math></b></p>  <p><b>Add 3 groups of 2 negative charges</b>  <b>The charge of the group is now <math>(-6)</math></b></p>
<p><u>Example 2:</u> <math>(-3) \times (-2) = ?</math></p>	<p><b>0 Charge</b></p>  <p><b><math>(-3) \times (-2)</math></b></p>  <p><b>To show multiplication of two negative integers, we have to take away 3 groups of 2 negative charges.</b></p>  <p><b>The result is a charge of positive 6.</b>  <b>So, <math>(-3) \times (-2) = 6</math></b></p>

**For more resources of multiplication and division of integers, see pages 144 – 146 of** Van de Walle, J., & Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson Education and Allyn & Bacon.

## Learning Task: What's Your Sign?

In 6<sup>th</sup> Grade, you were introduced to signed numbers (positive and negative) using a thermometer. In addition, you learned about absolute value (the distance a number is from zero on the number line), and opposites (two numbers that have the same absolute value).

In this unit, you are going to learn how to add, subtract, multiply and divide integers (whole numbers and their opposites) and rational numbers (fractions and decimals that are both positive and negative).

Number lines and counters are useful in demonstrating understanding of operations with integers.

1. Find four pairs of integers with a sum of 5. Explain your process.  
(Use positive integers only.)

Pair		Sum	Equation

2. Find four pairs of integers with a sum of 5. Explain your process.  
(Use at least one positive integer and at least one negative integer for each pair of integers.)

Pair		Sum	Equation

3. Find a pair of negative integers with a sum of 5. What do you notice about the result? Explain your findings.
  
4. What do you notice when adding integers?  
Look at your results from problems 1-3 to help you.

For the following examples, write an equation (show numerically) and draw a model using a number line or colored counters to help explain your answer.

5. Explain  $(-2) + 5$ .
  
  
  
  
  
  
  
  
  
  
6. Explain  $7 + 8$ .
  
  
  
  
  
  
  
  
  
  
7. Explain  $(-3) + (-4)$ .
  
  
  
  
  
  
  
  
  
  
8. Explain  $5 + (-8)$ .
  
  
  
  
  
  
  
  
  
  
9. Explain  $a + b$  if both  $a$  and  $b$  are positive numbers.
  
  
  
  
  
  
  
  
  
  
10. Explain  $(-a) + (-b)$  if  $(-a)$  and  $(-b)$  both represent negative numbers.
  
  
  
  
  
  
  
  
  
  
11. Explain  $a + (-b)$  if  $a$  represents any positive number and  $(-b)$  represents any negative number.

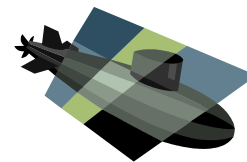
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12. Explain  $b + (-a)$  if  $b$  represents any positive number and  $(-a)$  represents any negative number.
13. Explain  $2 + 8 + (-7)$ .
14. Explain  $a + b + -c$  if  $a$  and  $b$  represent positive numbers and  $(-c)$  represents a negative number.





## Learning Task: Helicopters and Submarines



**INTRODUCTION:** You are an engineer in charge of testing new equipment that can detect underwater submarines from the air.

### **Part 1: The first three hours**

**During this part of the test, you are in a helicopter 250 feet above the surface of the ocean. The helicopter moves horizontally to remain directly above a submarine. The submarine begins the test positioned at 275 feet below sea level.**

- After one hour, the submarine is 325.8 feet below sea level.
- After two hours, the submarine dives another 23 feet.
- After three hours, the submarine dives again, descending by an amount equal to the average of the first two dives.

Make a table/chart with five columns (Time, Position of Submarine, Position of Helicopter, Distance between Helicopter and Submarine, and a Mathematical Sentence showing how to determine this distance) and four rows (start, one hour, two hours, three hours).

### **Part 2: The next three hours**

**The equipment in the helicopter is able to detect the submarine within a total distance of 750 feet.**

For each scenario, determine the maximum or minimum location for the other vehicle in order for the helicopter to detect the submarine; and write a mathematical sentence to show your thinking.

Determine the ordered pairs for these additional hours and include them on your graph.

- At the end of the fourth hour, the helicopter remains at 250 feet.
- At the end of the fifth hour, the submarine returns to the same depth that it was at the end of the third hour.
- At the end of the sixth hour, the submarine descends to three times its second hour position.

**Learning Task: Hot Air Balloons**

What happens to the balloon when...		Mathematically
<b>Add bags of gas</b>	<b>Balloon goes up</b>	3 bags of gas (+3) 10 bags of gas (+10)
<b>Add bags of sand</b>	<b>Balloon goes down</b>	3 bags of sand (-3) 10 bags of sand (-10)
<b>Subtract bags of gas</b>	<b>Balloon goes down</b>	Subtract 3 bags of gas – (+3) Subtract 10 bags of gas – (+10)
<b>Subtract bags of sand</b>	<b>Balloon goes up</b>	Subtract 3 bags of sand –(-3) Subtract 10 bags of sand – (-10)

When using hot air balloons to add or subtract integers, there are several important things to remember. They are:

- The first number indicates where the balloon starts.
- The sign tells you if you will be adding or subtracting something from the balloon. An addition sign tells you that you will be adding something to the hot air balloon and a subtraction sign tells you that you will be subtracting something from the balloon
- The second number tells you what you will add or subtract from the balloon (either bags of gas if the number is positive or bags of sand if the number is negative).

**Use a number line and a model of a hot air balloon. Model each problem and answer the questions that follow.**

**1.  $-3 + 6$**

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

**2.  $4 + (-7)$**

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

**3.  $3 + (-5)$**

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

**4.  $-2 + (9)$**

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

**5.** What do you think happens to the balloon if you take away sand instead of adding sand?

**6.** What do you think happens to the balloon if you take away gas bags instead of adding gas bags?

**7.  $6 - 9$**

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

8.  $-3 - (-9)$

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

9.  $-1 - (-3)$

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

10.  $-6 - 2$

- A. Where does the balloon start?
- B. Do you add or subtract something from the balloon?
- C. What do you add or subtract from the balloon?
- D. Where does the balloon end up?

**Model the following with your balloon and number line to answer each. Describe what you did with the air balloon.**

11.  $-2 + (-5)$

12.  $4 + (-9)$

13.  $-2 + 8$

14.  $5 - 9$

15.  $8 - (-6)$

## Performance Task: Debits and Credits

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money out of the account). The difference between debits/payments and deposits tells the value of the account. If there are more credits than debits, the account is positive, or “in the black.” If there are more debits than credits, the account is in debt, shows a negative cash value, or is “in the red.”

### **Situation #1:**

Use the ledger to record the information and answer the questions.

**Note:** On August 12, your beginning balance is \$0.00

1. On August 16, you receive a check from your Grandmother for \$40 for your birthday.
2. On August 16, you receive a check from your Parents for \$100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for \$23.42.
4. On August 18, you find \$5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for \$12.76.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

- A. What is your balance after five transactions?
- B. How much money did you deposit (show as a positive value)?
- C. How much money did you pay or withdraw (show as a negative value)?

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**Situation #2:**

Use the ledger to record the information and answer the questions.

**Note:** On May 5, your beginning balance is \$8.00

1. On May 6, you spent \$4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent \$3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received \$10 for cutting the neighbor's grass.
4. On May 8, you spent \$14.80 on a downloaded book for your Kindle.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

- A. What is your balance after four transactions?
  
  
  
  
  
  
  
  
  
  
- B. How much money did you deposit (show as a positive value)?
  
  
  
  
  
  
  
  
  
  
- C. How much money did you pay or withdraw (show as a negative value)?
  
  
  
  
  
  
  
  
  
  
- D. Can you really afford to spend \$14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of \$0?

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**Situation #3:**

Use the ledger to record the information and answer the questions.

**Note:** On July 4, your beginning balance is (-\$40).

Show, using at least eight transactions, a way you can have an ending account balance of more than \$145. You must include debit and credit amounts that have cents in at least five of your transactions. Your ledger must show both credits and debits.

Be sure to fill out the ledger as you go.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

## Learning Task: Multiplying Integers

You have recently practiced multiplying positive and negative integers with two-color counters. It is now your turn to model how to multiply. Below are “hints” to help you get started.

Remember that multiplication of whole numbers is often related to repeated addition. In elementary school, you learned that  $3 \cdot 5$  represents three sets or groups of five. Therefore you added three sets of five.

(+ + + + +) (+ + + + +) (+ + + + +)

Three sets of five equal fifteen. To apply this idea to multiplication involving positive and negative integers, let’s look at some sample problems.

1. When multiplying integers:

- The sign of the first factor tells us if we are “adding” or “taking away” sets.
- The first factor tells us how many sets.
- The sign of the second factor tells us what color (red or yellow) the groups consist of.
- The second factor tells us how many are in each set.

2. If the first factor is a negative integer, you have to “take away” sets of counters. To “take-away” counters, you will add as many zero-pairs as needed until you have enough counters to “take away”.

Model the following with two-color counters.

1.  $(2)(6)$

A. How is this read?

B. Model with your counters.

C. What is the solution?

2.  $(5)(3)$

A. How is this read?

B. Model with your counters.

C. What is the solution?



3.  $(-3)(2)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

4.  $(-4)(3)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

5.  $(2)(-4)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

6.  $(6)(-3)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

7.  $(-4)(-2)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

8.  $(-2)(-5)$

- A. How is this read?
- B. Model with your counters.
- C. What is the solution?

**Let's look to see if there are any patterns.**

1. Since multiplying two positive numbers is like adding groups, what happens when you add groups of positive numbers? Is the answer positive or negative?  
Is this **always** true?
2. Since multiplying a negative number and a positive number removes all positive numbers from the group, what is the result of adding the remaining negative numbers? Is the answer positive or negative? Is this **always** true?
3. Since multiplying a positive number and a negative number is like adding together groups of negative numbers, what is the result? Is the answer positive or negative?
4. Since multiplying a negative number and a negative number removes all negative numbers from the group, what is the result of adding the remaining positive numbers? Is the answer positive or negative? Is this **always** true?

## Learning Task: Multiplying Rational Numbers

You have recently practiced multiplying positive and negative integers on a number line. It is now your turn to model how to multiply. Below are “hints” to help you get started.

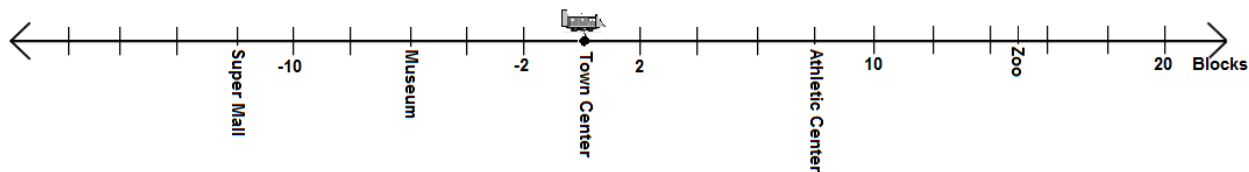
The rules for moving along the number line are as follows:

- Moving to the left or west means moving in a negative direction.
- Moving to the right or east means moving in positive direction.
- Time in the future is represented by a positive value.
- Time in the past is represented by a negative value.

### Multiplying Using the Number Line Model

Try these problems on your own.

In the city of Mathematica, there is a town center which attracts many visitors to the city. From the town center, a train takes visitors to different popular locations. The map shows a few favorite destinations that people like to visit. Answer the following problems about traveling around the city.



1. The train leaves Town Center traveling east at the speed of 2 blocks per minute. How many blocks will you be in 4 minutes? Where will you be in 4 minutes?
  
2. The train leaves the Town Center going west at 2 blocks per minute. What popular location will you arrive at in 6 minutes? How many blocks away from the Town Center will you be?
  
3. The train passes through the Town Center going east at 2 blocks per minute. Where was that train 3 minutes ago?

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4. You would like to take the train to the Zoo from the Town Center. How many minutes will this take if the train travels 2 blocks per minute? Write a math sentence to represent this scenario.
  
5. You waited 4 minutes for the train to arrive. The train was traveling west at 2 blocks per minute. Where was the train?

**Let's look to see if there are any patterns.**

1. When you moved east (right) and it was time in the future, two positive numbers represented the situation. What was the result on the number line when moving east and moving in the future occurred? Was the result of multiplying positive or negative? Is this **always** true?
  
2. When you moved east (right) and it was time in the past, a positive number and a negative number represented the situation. What was the result on the number line when moving east and moving in the past occurred? Was the result of multiplying positive or negative? Is this **always** true?
  
3. When you moved west (left) and it was time in the future, a negative number and a positive numbers represented the situation. What was the result on the number line when moving west and moving in the future occurred? Was the result of multiplying positive or negative? Is this **always** true?
  
4. When you moved west (left) and it was time in the past, two negative numbers represented the situation. What was the result on the number line when moving west and moving in the past occurred? Was the result of multiplying positive or negative? Is this **always** true?
  
5. What multiplication patterns can you see from each situation? Fill in the chart below according to the signs of the factors and products.

Factor	Factor	Product
+ number		+ number
	(-) number	(-) number
+ number		(-) number
	(-) number	+ number

### **Learning Task: Patterns of Multiplication and Division**

You have recently practiced dividing positive and negative integers on a number line. It is now your turn to model how to divide. Below are “hints” to help you get started.

When you divide, keep in mind these simple steps:

- Identify the dividend on the number line.
- Look at the divisor, is it positive (yellow with right arrow) or negative (red with left arrow).
- Determine how many times the divisor will have to move forward (+) or backward (-) to equal the dividend.
- The number of times it must move and the type of movement determine the answer.

Model the following on the number line.

1.  $8 \div 2$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

2.  $(-9) \div 3$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

3.  $(-10) \div 2$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

4.  $6 \div (-2)$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

5.  $8 \div (-2)$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

6.  $(-8) \div (-4)$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

7.  $(-4) \div (-2)$



- A. What is the dividend?
- B. What is the divisor?
- C. What is the solution and how did you find it?

**Let's look to see if there are any patterns.**

- 1. When given a positive integer as the dividend...
  - a. What was the result of dividing by a positive integer?
  - b. What was the result of dividing by a negative integer?
  
- 2. When given a negative integer as the dividend...
  - a. What was the result of dividing by a positive integer?
  - b. What was the result of dividing by a negative integer?

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3. Fill in each table below. Are the patterns the same as the multiplication patterns? Explain your findings.

<b>Multiplication Patterns</b>	<b>Factor</b>	<b>Factor</b>	<b>Product</b>
	+ number		+ number
		(-) number	(-) number
	+ number		(-) number
		(-) number	+ number

<b>Division Patterns</b>	<b>Dividend</b>	<b>Divisor</b>	<b>Divisor</b>
		+ number	+ number
	(-) number		(-) number
	+ number		(-) number
		(-) number	+ number



## **Learning Task: THE REPEATER vs. THE TERMINATOR**

Welcome to the first ever broadcast of the conflicts between THE REPEATER and THE TERMINATOR.

In corner 1, we have THE REPEATER. He is a  $152\frac{1}{3}$  pound fraction from Fayetteville, GA. THE REPEATER also likes to dress as  $152.\bar{3}$ . He enjoys working out via long division and *crushing* THE TERMINATOR

In corner 2, we have THE TERMINATOR. He is a  $152\frac{1}{4}$  pound fraction from Peachtree City, GA. THE TERMINATOR likes to disguise as 152.25 as a way to deceive his opponents. Like THE REPEATER, THE TERMINATOR enjoys working out with long division and *crushing* THE REPEATER.

In this task, you will change fractions to decimals to see if family members of REPEATER or TERMINATOR are bigger, stronger, and more likely to win against their opponents.

The chart below includes the first 20 unit fractions. For the 2<sup>nd</sup> column it is important that you remember the meaning of prime numbers and composite numbers. It is also important that you remember how to find the prime factorization of a number.

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UNIT FRACTION	PRIME FACTORIZATION OF DENOMINATOR	DECIMAL FORM	TERMINATES OR REPEATS
$\frac{1}{2}$	PRIME	.5	Terminates
$\frac{1}{3}$	PRIME	$.\bar{3}$	Repeats
$\frac{1}{4}$	$2 \cdot 2$		Terminates
$\frac{1}{5}$			
$\frac{1}{6}$		$.\bar{16}$	Repeats
$\frac{1}{7}$			
$\frac{1}{8}$	$2 \cdot 2 \cdot 2$		
$\frac{1}{9}$		$.\bar{1}$	
$\frac{1}{10}$			Terminates

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UNIT FRACTION	PRIME FACTORIZATION OF DENOMINATOR	DECIMAL FORM	TERMINATES OR REPEATS
$\frac{1}{11}$			
$\frac{1}{12}$			
$\frac{1}{13}$	Prime		
$\frac{1}{14}$		$.714285$	Repeats
$\frac{1}{15}$			
$\frac{1}{16}$			Terminates
$\frac{1}{17}$		$.0588235294117647$	
$\frac{1}{18}$	$2 \cdot 3 \cdot 3$		
$\frac{1}{19}$		$.052631578947368421$	
$\frac{1}{20}$			

1. What can you conclude about the types of rational numbers that have terminating decimals?
  
2. What can you conclude about the types of rational numbers that have repeating decimals?