Representing Fractions on a Number Line
Grade 3
Mathematics Formative Assessment Lesson

Designed and revised by Kentucky Department of Education Mathematics Specialists
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues, please contact the KDE team at:
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Created for the sole purpose of assisting teachers as they develop student understanding of Kentucky’s Core Academic Standard through the use of highly effective teaching and learning.

Not intended for sale.
Mathematical goals
This lesson is intended to help you assess how well students are able to:

- Understand a fraction as the quantity formed by 1 part when a whole is partitioned into equal parts.
- Represent a fraction on a number line diagram
- Understand two fractions are equivalent (equal) if they are the same point on a number line.
- Solve fraction word problems using the number line to represent solutions.

Common Core State Standards
This lesson asks students to select and apply Standards for Mathematical Content from across the grades, with the emphasis on:

**Number and Operations – Fractions 3.NF** (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Developing understanding of fractions as numbers.
3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts: understand a fraction a/b as the quantity formed by a parts of 1/b.

3.NF.2a.b. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3. NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

This lesson involves a range of Standards for Mathematical Practice from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Attend to precision
6. Look for and make use of structure
Overview

This lesson is structured in the following way:

- Before the lesson, students attempt the task individually. Review student work after students complete the initial task (Representing Fractions on the Number Line) and formulate questions for students to answer in order for them to improve their work.
- During the lesson the students will work in pairs to solve Six Friends Swimming using number lines – while the teacher notes misconceptions and successful solutions to be used in whole group discussion.
- In a whole class discussion, teachers facilitate a student discussion their responses to Six Friends Swimming.
- Students revisit initial task and try to improve their own responses.

Big Ideas Addressed in this lesson:

The goal is for students to see unit fractions as the basic building block of fractions, in the same sense that the number 1 is the basic building block of the whole numbers. Just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions.

On the number line, the whole is the unit interval, that is, the interval from 0 to 1, measured by length. Iterating this whole to the right, marks off the whole numbers, so that the intervals between consecutive whole numbers, from 0 to 1, 1 to 2, 2 to 3, etc., are all of the same length, as shown. Students might think of the number line as an infinite ruler.

Students sometimes have difficulty perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use as the unit the entire portion of the number line that is shown on the diagram. For example, indicating the number 3 when asked to show 3/4 on a number line diagram marked from 0 to 4.

The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so 5/3 is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to 1/3.

Linear models like the number line are closely connected to real world-measuring. The number line also emphasizes that a fraction is one number as well as its relative size to other numbers. The number line reinforces that there is always one more fraction to be found between two fractions.
Materials required

- Each student will need a copy of the initial assessment task, Representing Fractions on the Number Line.
- Each pair of students will need a copy of the Six Friends Swimming task.
- Optional: Whiteboards and markers
- Optional: String or clothesline rope for hands on Fraction Number Line. Number cards with fractions written on the cards.
- Optional: Fraction strips- pre-folded by students, fraction tiles, Cuisenaire rods,

Time needed

Approximately 20-30 minutes for the assessment task, one-hour or more for the lesson, and 20 minutes for the follow-up lesson where students revisit individual assessment task. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task: Representing Fractions on the Number Line

Have the students do the initial task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and identify areas of concern/need and target your follow-up instruction effectively.

Give each student a copy of Representing Fractions on the Number Line – Initial Task. Introduce the task briefly and help the class to understand the problem and its context. Students should have some prior experience working with number lines and whole numbers. This is also be an opportunity to help students make connections with fractions in the real world. Having students share any experiences with measurement and rulers may also help in giving some context to the initial task.

Possible instructions for students:

Spend 20-30 minutes working individually on this task.

Don’t worry if you can’t understand or do everything. There will be a lesson [tomorrow] that will help you improve your work.

Your goal is to be able to answer this question with confidence by the end of that lesson.

It is important that students are allowed to answer the questions without assistance. For struggling students, direct by paraphrasing or questioning, but do not complete the task for them.
Assessing students’ responses

Collect students’ responses to the task. Make some notes about what their work reveals concerning their current levels of understanding and their different problem solving approaches. This will help you prepare for the lesson and anticipate issues that may arise. If time allows you may write questions on each student’s work. If there are time constraints, select a few questions that will help the majority of students. These can be written on the board at the end of the lesson.

It is suggested that you do not score students’ work. Instead, help students progress by asking questions that focus attention on aspects of their work. Anticipating the different ways the task can be solved will help you in developing questions. Consider how your students mathematically interpret the task, use of correct and incorrect strategies to solve it, and how those strategies and interpretations relate to the mathematical ideas embedded in the task.

It is also suggested that you plan student pairings based on their work on this initial task - pairing students homogeneously (common understandings).
Common issues - Suggested questions and prompts:

<table>
<thead>
<tr>
<th>Common Misconceptions</th>
<th>Suggested Feedback Questions</th>
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<tbody>
<tr>
<td>Student plots points based on understanding fractions as whole numbers instead of fractional parts. For example: Students order fractions using the numerator:</td>
<td>Do $\frac{1}{2}$ and 1 mean the same thing?</td>
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<tr>
<td>Students order unit fractions by the denominator:</td>
<td>Tell me the difference between the size of $\frac{1}{4}$ and 1 whole?</td>
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<tr>
<td>Student sees the numbers in fractions as two unrelated whole numbers separated by a line.</td>
<td>Can you draw a picture of $\frac{1}{2}$? Can you draw $\frac{1}{4}$? Which is closer to 1 whole?</td>
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<tr>
<td>Students do not understand that when partitioning a whole or a fraction into unit fractions, the intervals must be equal.</td>
<td>When I show this fraction $\frac{2}{3}$, what does it mean? Does it mean 2 and 3 separated by a line?</td>
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<tr>
<td>Student does not understand the importance of the whole of a fraction and identifying it. For example, students may use a fixed size of $\frac{1}{4}$ based on the manipulatives used or previous experience with a ruler.</td>
<td>What can you tell me about all the [fourths] of a whole?</td>
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</table>
|                                                                                       | Is $\frac{1}{4}$ inch the same as $\frac{1}{4}$ of [this whole]?


### Student does not count correctly on the number line. For example, students may count the hash mark at zero as the first number in the sequence:

Where are the parts of your whole? How many parts are there?

### Student does not understand there are many fractions less than 1.

Could you place one half on the number line? Where would you place one fourth? What do you notice when you do this? Can you do it for other fractions?

### Student does not understand fractions can be greater than 1.

Can you count using fractions like you do with whole numbers? How would you count if you were counting by 1’s, 2’s, and ½’s? Can I have 3/2 cookies, hours, days, inches? Would that be less than or more than 1 [cookie]?
Suggested lesson outline

Collaborative Activity: Six Friends Swimming Task

Have students work in pairs to solve the Six Friends Swimming task. There are two versions of the Six Friends Swimming Task. Version B is slightly more challenging. Based on your assessment of the initial student task, you may want some pairs to start with Version A, while other pairs start with Version B. You may want all pairs to start with Version A, then move to Version B. Students may need to see a photograph of swimming lanes in a pool for background information.

Monitoring Student Groups

As partners work on solving the task pay close attention to students’ mathematical thinking and solution strategies by circulating around the classroom. According to Lampert (2001, p. 140), paying close attention to what students do as they work makes it possible “to use my observations to decide what and who to make focal” during the whole group discussion that follows. As you move about the room observing the work of the partners, note their solutions and select those responses that will help you meet the targets of the lesson. You will also need to decide the sequence of responses during the whole-group discussion.

To further the monitoring process, the teacher can create a list of solutions before teaching the lesson to anticipate what students will produce. Working with other teachers to anticipate possible student responses is an important part of this process.

Whole Group Discussion

Bring student pairs together as a whole group to share how they solved Six Friends Swimming. For pairs to share their solutions you could use a smartboard or document camera, a clothesline number line, or floor number line. The whole group discussion is a great time for students to use the mathematical practice, Construct viable arguments and critique the reasoning of others. The discussion offers students the opportunity to learn from each other and for you to address some of the common misconceptions observed in the initial task and Six Friends Swimming. Students should be expected to use this time to compare their solutions, discuss misconceptions and eventually evaluating their own responses based on correct answers. As part of the culture in your mathematics classroom students need to feel safe to share their solution strategies and ask questions of the teacher and each other.

Were there certain problems that were more difficult? Did you use the number line, or did you use a different method to solve the problems? Can different fractions represent the same place on the number line?

Could you use what you know about whole numbers to help you solve the problems?

You could introduce the fraction strips as another way to represent fractions used in the number line. How could you use the fraction strips to help you solve the Six Friends Swimming problems? How are the fraction strips similar/different than the number line?
Have students think about where they are at this point in the process? What have they learned? What is still giving them trouble? What do I know? What do I need to work on?

**Improving individual solutions to the assessment task**

Return the student’s original assessment, Representing Fractions on a Number Line. You will also want to provide them with a copy of Representing Fractions on a Number Line – Revisit Task.

Possible instructions for students:

*Look at your original responses and think about what you have learned during this lesson.*  
*Using what you have learned, is there anything you want to change?*

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.
Representing Fractions on a Number Line - Initial Task

1. Draw points on the number line below for ¼, 2/4, 3/4, 4/4. Label the points. Be as exact as possible.

2. Draw points on the number line below for ½, 1/3, 1/4, 1/6, 1/8. Label the points. Be as exact as possible.

3. Label the points on the number line below.

4. Anna drew a point on the number line for 1. Do you agree or disagree with where Anna drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.

5. Fred drew a point on the number line for 1. Do you agree or disagree with where Fred drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.
**Six Friends Swimming-Version A**

Six friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they have swam.

Addison - 1/3  
Caleb - ½  
Benjamin – 1/4  
Mary- 1/6  
Ellen – 1/8

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

**Addison**

![Addison's position on the number line]

**Benjamin**

![Benjamin's position on the number line]

**Caleb**

![Caleb's position on the number line]

**Mary**

![Mary's position on the number line]

**Ellen**

![Ellen's position on the number line]
Six Friends Swimming-Version A

Use the lane markers (number lines) to solve the problems with a partner.

1. Benjamin is training for the mile swim. He swims $\frac{1}{4}$ mile the first day. If he swims $\frac{1}{4}$ mile a day, how many days will take him to swim 1 mile?

![Number line](image)

2. Ellen is training for the mile swim. She swims $\frac{1}{8}$ mile each day. On the first day, she swims $\frac{1}{8}$ mile. How far did Ellen swim in 2 days? How far will Ellen swim in 4 days?

![Number line](image)

3. On Wednesday, Caleb said he swam $\frac{1}{2}$ mile. Mary swam $\frac{3}{6}$ miles. Ellen swam $\frac{4}{8}$ miles. Addison swam $\frac{2}{4}$ miles. Caleb said, “we all swam the same distance and we’re tied.” Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.

![Number line](image)
Six Friends Swimming-Version A

4. Caleb swam \( \frac{1}{2} \) mile a day for 3 days. At the end of 3 days Caleb told his friends he swam \( \frac{3}{2} \) miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as \( \frac{3}{2} \)?

**Six Friends Swimming-Version B**

Six friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they have swam.

Addison - $\frac{3}{4}$  
Caleb - $\frac{1}{2}$  
Benjamin - $\frac{5}{6}$  
Mary - $\frac{5}{8}$  
Ellen - $\frac{2}{3}$

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

Addison

![Addison Number Line](image)

Benjamin

![Benjamin Number Line](image)

Caleb

![Caleb Number Line](image)

Mary

![Mary Number Line](image)

Ellen

![Ellen Number Line](image)
**Six Friends Swimming-Version B**

Use the lane markers (number lines) to solve the problems with a partner.

1. Addison is training for a swimming race. If he swims $\frac{1}{4}$ mile a day, how many days will it take him to swim $\frac{1}{2}$ mile?

2. Mary swims $\frac{1}{8}$ mile each day. How far will Mary swim in 12 days?

3. On Wednesday, Caleb said he swam 2 miles. Benjamin swam $\frac{6}{3}$ miles. Mary swam $\frac{4}{2}$ miles. Addison swam $\frac{6}{4}$ miles. Caleb said, “we all swam the same distance and we’re tied.” Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.
Six Friends Swimming-Version B

4. Caleb swam 1/6 mile a day for 8 days. At the end of 8 days Caleb told his friends he swam 4/3 miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as 4/3?

Remember: Grade 3 KCAS limits denominators to 2, 3, 4, 6, and 8

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Representing Fractions on a Number Line - Revisit Task

1. Draw points on the number line below for $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$. Label the points. Be as exact as possible.

2. Draw points on the number line below for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$. Label the points. Be as exact as possible.

3. Label the points on the number line below.

4. Anna drew a point on the number line for 1. Do you agree or disagree with where Anna drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.

5. Fred drew a point on the number line for 1. Do you agree or disagree with where Fred drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.
Strong Model of Student Work:

1. Draw points on the number line below for $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$. Label the points. Be as exact as possible.

2. Draw points on the number line below for $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{8}$. Label the points. Be as exact as possible.

3. Label the points on the number line below.

4. Anna drew a point on the number line for 1. Do you Agree or Disagree with where Anna drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.
5. Fred drew a point on the number line for 1. Do you Agree or Disagree with where Fred drew the point? If you disagree, draw the point where you think it should be. If you agree, explain why you agree.

Fractions are split into equal groups. \( \frac{5}{3} \) is \( \frac{2}{3} \) away from 1.
Solutions:

**Six Friends Swimming-Version A**

Six friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they have swam.

Addison - 1/3  
Caleb - 1/2  
Benjamin - 1/4

Mary - 1/6  
Ellen - 1/8

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

Addison

Benjamin

Caleb

Mary

Ellen
Six Friends Swimming-Version A

Use the (number lines) on page 12 to solve the problems with a partner.

1. Benjamin is training for the mile swim. He swims \( \frac{1}{4} \) mile a day. How many days will take him to swim 1 mile? \( 4 \) days

2. Ellen is training for the mile swim. She swims \( \frac{1}{8} \) mile each day. On the first day, she swims \( \frac{1}{8} \) mile. How far did Ellen swim in 2 days? How far will Ellen swim in 4 days?

3. On Wednesday, Caleb said he swam \( \frac{1}{2} \) mile. Mary swam \( \frac{3}{6} \) miles. Ellen swam \( \frac{4}{8} \) miles. Addison swam \( \frac{2}{4} \) miles. Caleb said, “we all swam the same distance and we’re tied.” Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.

So \( \frac{1}{2}, \frac{3}{6}, \frac{4}{8}, \) and \( \frac{2}{4} \) all represent the same distance.
Six Friends Swimming-Version A

4. Caleb swam ½ mile a day for 3 days. At the end of 3 days Caleb told his friends he swam 3/2 miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as 3/2?

![Number Line Diagram]

* Students may name any equivalent fraction for 3/2, such as 6/4, 12/8.

* Note: Students in 3rd grade are not expected to use mixed numbers.

Six Friends Swimming-Version B

Six friends are swimming in a 1 mile race at the lake. Each swimmer has a lane marker to lead them to the finish line. The fractions tell how much of the 1 mile distance they have swam.

Addison - 3/4  Caleb - 1/2  Benjamin - 5/6
Mary - 5/8  Ellen - 2/3

Place each swimmer on their lane marker (number line) to show where they are between the start and finish. Be as exact as possible when you show where each swimmer is in the race.

Addison

Benjamin

Caleb

Mary

Ellen
Six Friends Swimming-Version B

Use the (number lines) to solve the problems with a partner.

1. Addison is training for a swimming race. If he swims $\frac{1}{4}$ mile a day, how many days will it take him to swim $\frac{3}{4}$ mile?

   2 days

2. Mary swims $\frac{1}{8}$ mile each day. How far will Mary swim in 12 days?

   $\frac{12}{8}$ miles

3. On Wednesday, Caleb said he swam 2 miles. Benjamin swam $\frac{6}{3}$ miles. Mary swam $\frac{4}{2}$ miles. Addison swam $\frac{6}{4}$ miles. Caleb said, “we all swam the same distance and we’re tied.” Is what Caleb said true? Prove that whatever Caleb is saying is true or false. Use the lane markers (number lines) to show your work.

   No, Addison swam less.
Six Friends Swimming-Version B

4. Caleb swam $\frac{1}{6}$ mile a day for 8 days. At the end of 8 days Caleb told his friends he swam $\frac{4}{3}$ miles all together. Did Caleb use the correct fraction? Show your work and explain if you think Caleb is correct or incorrect. Is there another fraction Caleb could use that means the same thing as $\frac{4}{3}$?